Tuned Mass Dampers in Passenger Cars

Anand S [a] *, Kalyan Raj AH[a],Kartik B[a], Dhanushkodi DM[b]

[a]Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai
[b]TechPassion Technologies Pvt. Ltd.

Abstract

It is very well known that tuned mass dampers are engineered to minimise the vibrations in buildings in earthquake prone areas and in some cases machinery [1]. This paper evaluates the utility of Tuned Mass Dampers in passenger cars. For the same suspension system, addition of a tuned mass was found to reduce the transmissibility by a considerable amount. Addition of a tuned mass also helps in decoupling, to an extent, of road holding and ride comfort. The vibrations of the chassis are taken up by the tuned mass which improves the ride comfort, while road handling is taken care of by the damping-stiffness combination of the existing suspension. Hence, the existing suspension can be designed with more focus on road handling. The effect of changing various parameters like damping of suspension and tuned mass, stiffness of the springs etc. were studied.

*Corresponding author. Tel.:+91 9444376726, +91 44 43514140; fax: +91 44 43514164. E-mail address: anands@iitm.ac.in (Anand S).

Notations

List of notations used in this paper:

M	$=$	Mass of the main body (sprung mass in the case of the quarter car and half car models)
m	$=$	Mass of the TMD
m_w	$=$	Unsprung Mass
k	$=$	Spring stiffness of spring attached to TMD
K	$=$	Spring stiffness of spring attached to main body
k_w	$=$	Spring stiffness of spring attached to unsprung mass
C	$=$	Damping associated with main body
c	$=$	Damping associated with TMD
c_w	$=$	Damping associated with unsprung mass
ω	$=$	input frequency
Ω_n	$=$	$\sqrt{K/M}$ (Natural frequency of the main body)
ω_a	$=$	$\sqrt{k/m}$ (Natural frequency of the TMD)
ω_w	$=$	$\sqrt{k_w/m_w}$ (Natural frequency of the unsprung mass)
ζ_1	$=$	$c/2m\omega_a$
ζ_2	$=$	$C/2M\Omega_n$
ζ_w	$=$	$c_w/2m_w\omega_w$
g	$=$	ω/Ω_n
f	$=$	ω_a/Ω_n
f_w	$=$	ω_w/Ω_n
μ	$=$	m/M
μ_w	$=$	m_w/M

1 Introduction

Tuned mass dampers are used quite widely in various applications to reduce vibrations in a system, including in buildings to reduce vibrations due to earthquakes and winds, in machines, in engine shafts etc. The basic idea of a tuned mass damper is to introduce a new mass with the same natural frequency of the original system.

This leads to significant reduction in amplitude of vibration at the natural frequency of the original system, though it introduces two new resonant frequencies. It is seen that the two peaks of the new system are lower than the single peak which was there before. This is because more energy is being dissipated through the damper which connects the tuned mass. The tuning enables the energy to be transferred effectively. Thus when tuned correctly the two peaks will be well below the single original peak.

For systems where the input is of a very specific frequency, the tuned mass is very effective. But even for systems subjected to broad band input, the tuned mass is useful, since the two new peaks that are introduces can be tuned to be much lower than the single peak of the original system. The mass, stiffness and damping of the new mass has to be tuned to achieve a state where the maximum amplitude reached by the system is minimised.

A system like an automobile, in particular a passenger car, is one of the systems that is subjected to a broadband random input, usually modelled as white noise input. For this system, a comprehensive modelling and optimisation is required to find the appropriate values of mass, stiffness and damping for which the maximum amplitude of vibration is minimised.

In this paper, quartercar, halfcar models are analysed with a tuned mass, and the parameters are optimised to decrease the transmissibility of the system without modifying the existing suspensions. The tuned masses are used to reduce the pitch and bounce, with more weight on reducing pitch, since it is

the motion that causes the most discomfort [2].

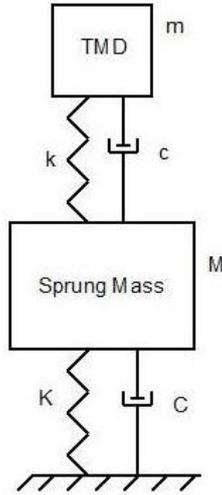## 2 General properties of tuned mass damper

Figure 1: TMD comparison of systems with different damping

As discussed in [1], for a single mass spring-damper system similar to one shown in figure 1 with a tuned mass attached to it, the transfer function varies with g as shown in Figure 2 for different values of damping. The first thing to note in the graph in figure 2 is that the introduction of a tuned mass damper
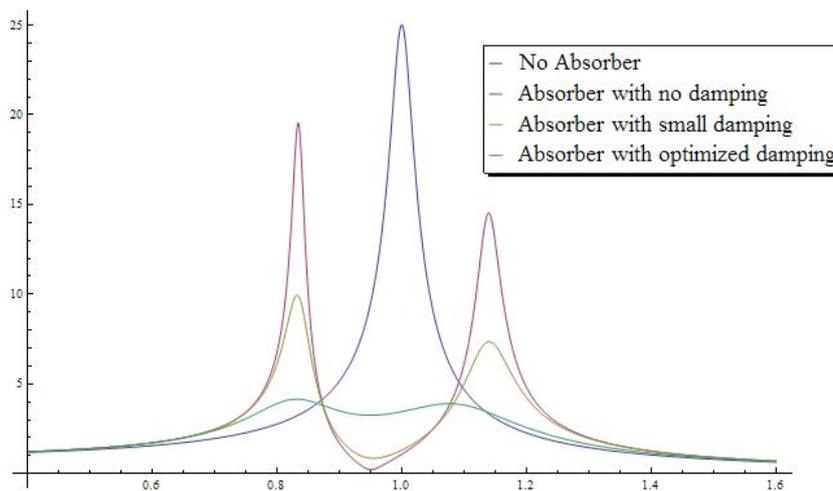

Figure 2: TMD comparison of systems with different damping

to the system splits the original single resonance peak to two peaks of slightly lesser amplitudes, while at the original resonant frequency, the amplitude is nearly zero. Also, both for very low and very high values of the tuned mass damping – ζ_1, the magnitude of the transfer function is quite high at two values of g. A similar thing happens when f (which is a function of k) is varied. When optimal values of f and ζ_1 are chosen, both the peaks become equal and the maximum amplitude of the transfer function at any frequency is greatly reduced. This in effect is the consequence of adding a tuned mass damper.

3   Analysis of the effect of mass damper in a Quartercar model

Quarter car model is the most basic method used to model the vibration isolation properties of a car suspension [2]. The effect of addition of a tuned mass damper to a quarter car is analysed here.
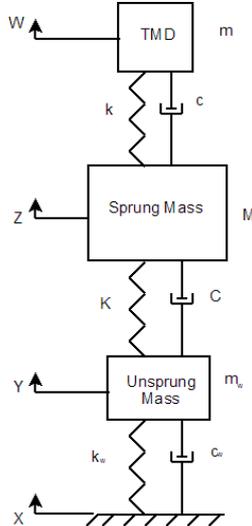

Figure 3: Quarter car model with TMD

In the following derivation, the Quarter car model with a tuned mass damper used is as shown in Figure 3. This model was used to get the transfer function for bounce which was taken to be Z/X. The equations of the system in frequency domain are written down. Nondimensionalising the equation and solving for Z and X we get Z/X as follows:

$$G(g) = \left(f_w \left(f^2 - g^2 + 2if g\zeta_1\right)\left(-i + 2g\zeta_2\right)\left(-i f_w + 2g\zeta_w\right)\mu_w\right) / \left(g^2\left(-g^2 + f^2(1 + \mu)\right) - \right.$$
$$\left(g^2 - g^4 + f^2\left(-1 + g^2(1 + \mu)\right)\right)\left(g^2 - f_w\left(f_w + 2ig\zeta_w\right)\right)\mu_w + 2ig\zeta_2\left(g^2\left(-g^2 + f^2(1 + \mu)\right)\right.$$
$$+ \left(f^2 - g^2\right)\left(g^2 - f_w\left(f_w + 2ig\zeta_w\right)\right)\mu_w\right) + 2fg\zeta_1\left(-g^2(1 + \mu)\left(-i + 2g\zeta_2\right) - i\left(-1 + g^2(1 + \mu) - 2ig\zeta_2\right)\right.$$
$$\left.\left.\left(g^2 - f_w\left(f_w + 2ig\zeta_w\right)\right)\mu_w\right)\right)$$

This transfer function is a complex function, and we are concerned here with the absolute value of this function.

4   Optimising the parameters of the mass damper for the Quartercar model

The preceding transfer function is used to optimise the the values of f and ζ_1 for given values of $\zeta_2, \zeta_w, f_w, \mu, \mu_w$. From the obtained values of f and ζ_1 the values of k and c can be calculated. The value of μ is not optimised because there are no opposing forces involved, and higher the value of μ the better the reduction in the magnitude of the transfer function. So, for practical considerations, the value of μ is taken to be 0.04.

The optimisation is done using an ordered search algorithm. The values of f, ζ_1 are varied systematically, f from 0.1 to 2 and ζ_1 from 0.01 to 0.2. For each of these sets of values, the global maximum of the absolute value of transfer function $G(g)$ is found and it is seen when this value turn out to be

minimum. Put concisely we are trying to minimize the global maximum of $|G(g)|$. Thus we get a one set of values for f and ζ_1 for which the stated objective is achieved. These are the optimized values and now we consider the mass attached to be 'tuned'.
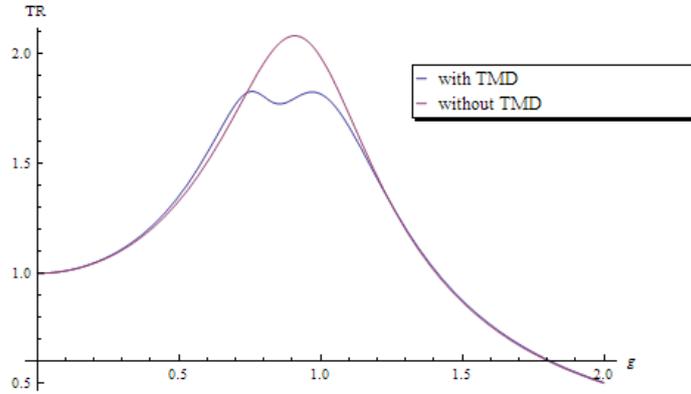

Figure 4: Quarter car transmissibility comparison

The figure 4 shows the effect of adding a TMD to a quarter car. As can be seen, the transmissibility is reduced by a considerable amount.

Transmissibility is 1.816 for the case with TMD, and 2.086 for the case without. This implies a reduction of 12.94% in transmissibility of this quarter car.

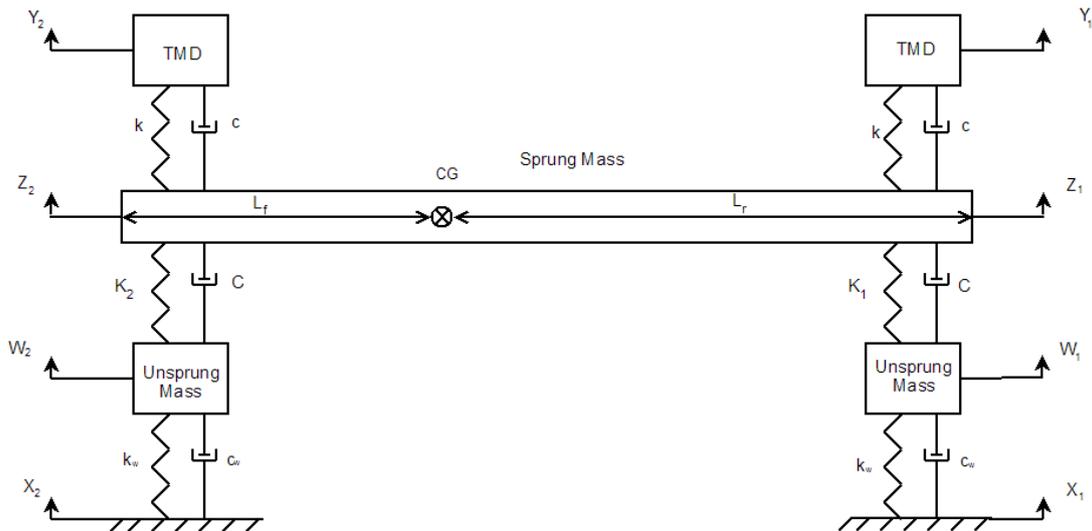# 5 Analysis of the effect of mass damper in a Halfcar model– Reducing pitch vibrations

Figure 5: Half car model with TMD

Quartercar models can describe only one kind of motion i.e bounce.A halfcar model is used so that pitching motion can be studied. This half car model employs two tuned masses, one at the front and one at the rear end of the car to attenuate pitching motion. We ensure that the total added tuned mass is only 4 percent of the sprung mass. For generality the C.G is shifted towards the front as would be

in a Front Wheel Drive. Also differential spring rates for front and back are considered in the main suspension. The differential spring rates itself reduce pitching substantially [3]. Thus, the model shown in figure 5 is a reasonably generalized model of a half car.

The Transfer function for pitching motion is taken as $(Z_2 - Z_1)/X_2$. We give X_2 as a delta function input and set X_1 to 0 so that we can characterize pitching motion. This transfer function is got from solving the equation (in frequency domain) $A.X = B$ where

$$
A = \begin{pmatrix}
\left(1 + \beta - g^2\beta + 2ig(1+\beta)\zeta_2\right)/1+\beta & \left(-g^2 + \delta + \beta\delta + 2ig(1+\beta)\zeta_2\right)/1+\beta & -g^2\mu \\
L_r\left(1 - g^2\alpha(1+\beta) + 2ig\zeta_2\right) & -L_r\left(-g^2\alpha(1+\beta) + \beta\delta + 2ig\beta\zeta_2\right) & -g^2\mu L_r \\
-f\mu\left(f + 2gi\zeta_1\right) & 0 & \mu\left(f^2 - g^2 + 2ifg\zeta_1\right) \\
0 & -f\mu\left(f + 2gi\zeta_1\right) & 0 \\
-1 - 2gi\zeta_2 & 0 & 0 \\
0 & -\delta - 2gi\zeta_2 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
-g^2\mu & -1 - 2gi\zeta_2 & -\delta - 2gi\zeta_2 \\
g^2\beta\mu L_r & -L_r\left(1 + 2gi\zeta_2\right) & \beta L_r\left(\delta + 2gi\zeta_2\right) \\
0 & 0 & 0 \\
\mu\left(f^2 - g^2 + 2ifg\zeta_1\right) & 0 & 0 \\
0 & 1 + 2ig\zeta_2 + \left(-g^2 + f_u^2 + 2igf_u\zeta_u\right)\mu_u & 0 \\
0 & 0 & \delta + 2ig\zeta_2 + \left(-g^2 + f_u^2 + 2igf_u\zeta_u\right)\mu_u
\end{pmatrix}
$$

$$
B = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
f_u X_1 \left(f_u + 2gi\zeta_u\right)\mu_u \\
f_u X_2 \left(f_u + 2gi\zeta_u\right)\mu_u
\end{pmatrix}
$$

$$
X = \begin{pmatrix}
Z_1 \\
Z_2 \\
Y_1 \\
Y_2 \\
W_1 \\
W_2
\end{pmatrix}
$$

6 Optimising the parameters of the mass damper for the Halfcar model

Just as we did for the quarter car model we take the absolute value of the complex transfer function for halfcar and find at what g its global maximum lies (The Resonance Frequency). We try to minimize this value by cycling through the parameters we want to optimize systematically as indicated before. We use the same ordered search algorithm, the only difference being that the transfer function is more complicated. The values used here are as follows: $\zeta_2 = 0.2371, \zeta_u = 0.0612, \alpha = 1.11, f_u = 12.9099, \mu = 0.02, \mu_u = 0.06, \beta = 0.5633, \delta = 0.7$. Using these values and optimising, we get $\zeta_1 = 0.16, f = 0.546$,

The figure 6 shows the effect of adding a TMD to a half car. As can be seen, the transmissibility is reduced by a considerable amount.

Transmissibility is 3.090 for the case with TMD, and 3.528 for the case without. This implies a reduction of 12.415% in transmissibility of this half car.

It was also checked to see if bounce is affected adversely by adding the mass (since we tuned only for pitching motion). Bounce remained nearly unaffected by the addition of these masses. Pitching motion is sensed to be far more disturbing than bouncing motion by the human body [2].
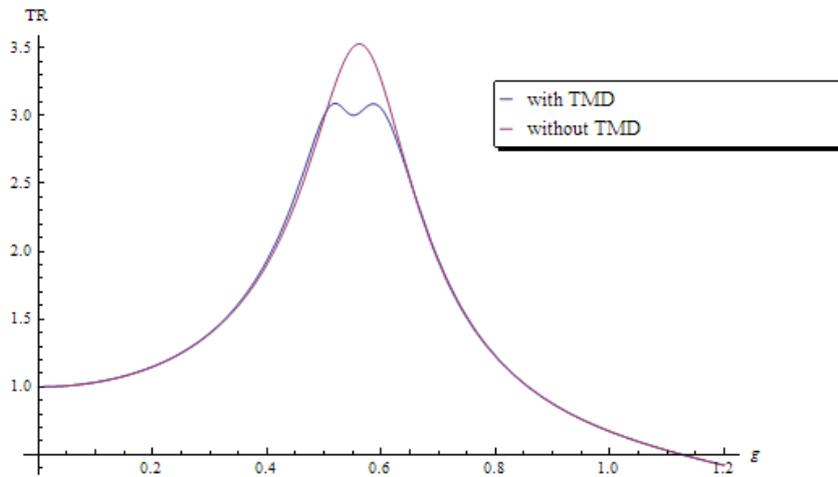
Figure 6: Half car transmissibility comparison

7 Conclusion

The objective of adding the tuned mass is to achieve increased ride comfort without conflicting with road holding requirements. Also the addition of mass (around 4 percent of sprung mass) seems to be a relatively small price to pay for the increased ride comfort that we get.

References

[1] Den Hartog, JP.,*Mechanical Vibrations (4th edition)*, McGraw-Hill, New York;(1956).

[2] Gillespie TD.,*Fundamentals of Vehicle Dynamics*, SAE Inc., Warrendale;(1992).

[3] William F. Milliken and Douglas L. Milliken. *Chassis Design: Principles and Analysis*,Based on previously unpublished technical notes by Maurice Olley,SAE Inc.;(2002).